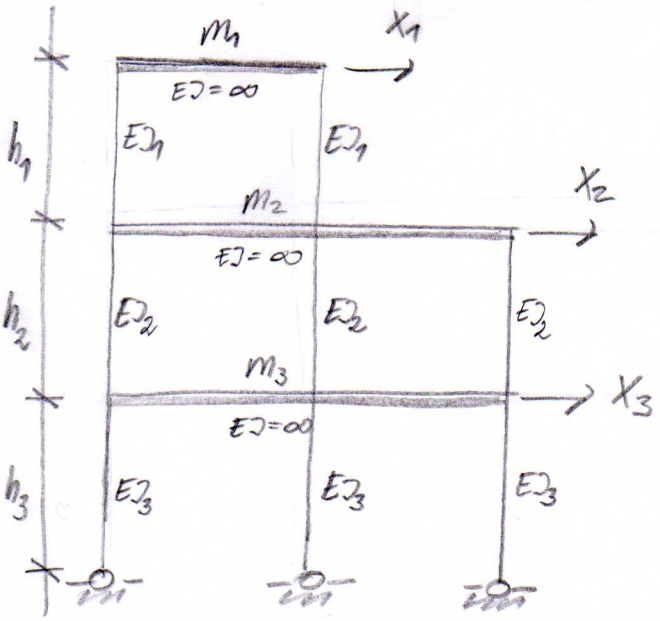
ODREĐIVANJE VIŠIH TONOVA MATRIČNOM ITERACIJOM



zadano je:

$m_1 = 12\,t$; $m_2 = 24\,t$ $m_3 = 32\,t$

$EJ_1 = 20250\ kNm^2$ (30/30)

$EJ_2 = 48000\ kNm^2$ (30/40)

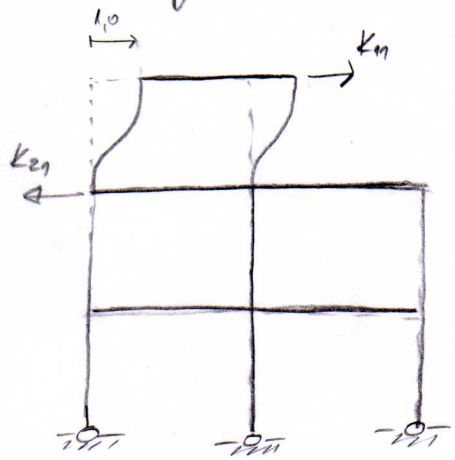
$EJ_3 = 64000\ kNm^2$ (40/40)

$h_1 = 2{,}5\ m$

$h_2 = 3\ m$

$h_3 = 4\ m$

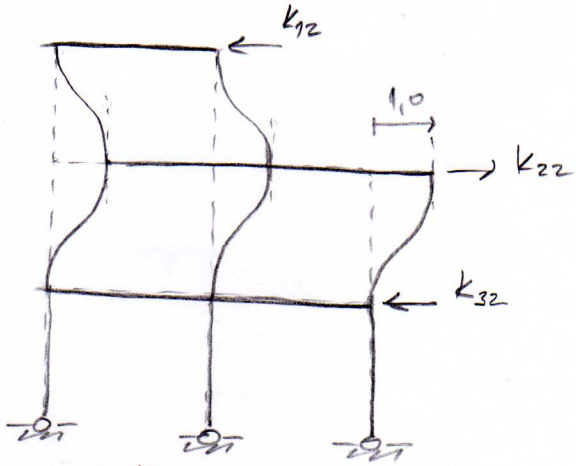
ODREĐIVANJE MATRICE KRUTOSTI



$$k_{11} = 2 \cdot \frac{12EJ_1}{h_1^3} = \frac{24EJ_1}{h_1^3} = 31104\ kN/m$$

$$k_{21} = -2 \cdot \frac{12EJ_1}{h_1^3} = -31104\ kN/m$$

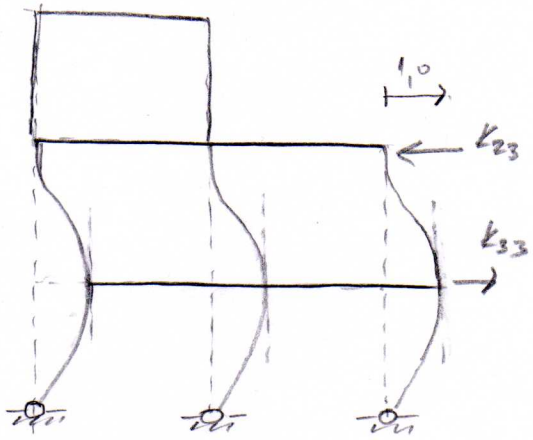
$$k_{31} = \emptyset$$



$$k_{12} = -2 \cdot \frac{12EJ_1}{h_1^3} = -31104\ kN/m$$

$$k_{22} = \frac{24EJ_1}{h_1^3} + \frac{36EJ_2}{h_2^3} = 95104\ kN/m$$

$$k_{32} = -\frac{36EJ_2}{h_2^3} = -64000\ kN/m$$



$$k_{13} = 0$$

$$k_{23} = -\frac{36EJ_2}{h_2^3} = -64000\ kN/m$$

$$[M] = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix}$$

$$[k] = \begin{bmatrix} 31104 & -31104 & 0 \\ -31104 & 95104 & -64000 \\ 0 & -64000 & 73000 \end{bmatrix}$$

MATRICA FLEKSIBILNOSTI

$$[F] = [k]^{-1} = \frac{1}{9000} \cdot \begin{bmatrix} \frac{2471}{1728} & \frac{73}{64} & 1 \\ \frac{73}{64} & \frac{73}{64} & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1,5888 & 1,2673 & 1,1111 \\ 1,2673 & 1,2673 & 1,1111 \\ 1,1111 & 1,1111 & 1,1111 \end{bmatrix} \cdot 10^{-4}$$

MATRIČNA ITERACIJA PREKO MATRICE KRUTOSTI

Problem slobodnih oscilacija:

$$[M]\{\ddot{x}\} + [k]\{x\} = \{0\}$$

→ možemo pretpostaviti da će pomaci za vrijeme slobodnih oscilacija biti opisani harmonijskim funkcijama, te znam. da amplitude pomaka mogu biti različite na pojedinim mjestima

pretp. da pomaci ovise o ← mjestu na kont. preko vektora $\{v\}$, i da se na svim mjestima sinhrono mijenjaju u vremenu s frek. ω

$$\{x\} = \{v\} \cdot \sin \omega t$$

$$\{\ddot{x}\} = -\omega^2 \{v\} \sin \omega t$$

$$-\omega^2 [M]\{v\} + [k]\{v\} = \{0\}$$

$$[k] \cdot \{v\} = \omega^2 [M]\{v\} \quad / \cdot [M]^{-1}$$

$$[M]^{-1}[k]\{v\} = \omega^2 [I] \cdot \{v\}$$

→ iteracijom preko matrice krutosti prvo dobivamo najnižu formu oscilacija

dinamička matrica ← $[D]$

$\omega^2 = \lambda$

$$[D] \cdot \{v\} = \lambda \cdot \{v\}$$

$$[D] = [M]^{-1}[k] = \begin{bmatrix} 2592 & -2592 & 0 \\ -1296 & 3962,67 & -2666,67 \\ 0 & -2000 & 2281,25 \end{bmatrix}$$

Iteracija $[D]\{v_3\} = \lambda \{v_3\}$

			1	-0,654	-0,686	-0,701	-0,708	-0,710
			-1	1,0	1,0	1,0	1,0	1,0
			1	-0,540	-0,517	-0,510	-0,508	-0,507
2592	-2592	0	5184	-4287,44	-4369,83	-4409,92	-4426,01	-4432,41
-1296	3962,67	-2666,67	-7925,34	6259,92	6230,51	6232,52	6233,52	6233,92
0	-2000	2281,25	4281,25	-3232,33	-3179,63	-3164,19	-3158,17	-3155,78
		λ_3	-7925,34	6259,92	6230,51	6232,52	6233,52	6233,92

Treća vlastita frekvencija $\lambda_3 = \omega_3^2 = 6233{,}92 \rightarrow \omega_3 = 78{,}955$ r/s

Treći vlastiti vektor: $v_3 = \begin{Bmatrix} -0{,}710 \\ 1{,}0 \\ -0{,}507 \end{Bmatrix}$

tj. $\omega_1 = 11{,}2228$
$\omega_2 = 49{,}7572$
$\omega_3 = 78{,}9869$

DRUGA FORMA

Uvjet ortogonalnosti vlastitih vektora: $\{v_2\} \perp \{v_3\}$

$$\{v_2\}^T [M] \{v_3\} = \{0\}$$

$$\{v_{12} \; v_{22} \; v_{32}\} \cdot \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \cdot \begin{Bmatrix} -0{,}710 \\ 1{,}0 \\ -0{,}507 \end{Bmatrix}$$

$$v_{12} = 2{,}8169 \cdot v_{22} - 1{,}904 \, v_{32}$$

Napisano u matričnom obliku

$$\underbrace{\begin{bmatrix} 0 & 2{,}8169 & -1{,}904 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{[S_1]} \begin{Bmatrix} v_{12} \\ v_{22} \\ v_{32} \end{Bmatrix} = \begin{Bmatrix} v_{12} \\ v_{22} \\ v_{32} \end{Bmatrix}$$

$[S_1]$ – matrica ekvivalencije

$$[D_2] = [\Delta] \cdot [S_1] = \begin{bmatrix} 0 & 4709{,}4 & -4935{,}17 \\ 0 & 311{,}96 & -199{,}08 \\ 0 & -2000 & 2281{,}25 \end{bmatrix}$$

Iteracija $[D_2] \cdot \{v_2\} = \lambda \{v_2\}$

	1	1,0	1,0	1,0	1,0
	1	0,053	0,043	0,042	0,042
	-1	-0,444	-0,458	-0,459	-0,459
0 4709,4 -4935,13	9644,67	2440,27	2464,74	2466,02	2466,02
0 311,96 -199,08	511,04	104,90	104,67	104,66	104,66
0 -2000 2281,25	-4281,3	-1118,6	-1131,7	-1132,4	-1132,4

Druga vlastita frekvencija $\lambda_2 = \omega_2^2 = 2466{,}02 \rightarrow \omega_2 = 49{,}66$

Drugi vlastiti vektor $v_2 = \begin{Bmatrix} 1{,}0 \\ 0{,}042 \\ -0{,}459 \end{Bmatrix}$

Prvi oblik oscilacija

→ uvjet ortogonalnosti vlastitih vektora $\{v_1\}^T \perp \{v_2\}$ i $\{v_1\}^T \perp \{v_3\}$

$$\{v_1\}^T [M] \{v_2\} = \{0\}$$

$$\{v_{11}\ v_{21}\ v_{31}\} \cdot \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \cdot \begin{Bmatrix} 1{,}0 \\ 0{,}042 \\ -0{,}459 \end{Bmatrix} = 0$$

$$12 v_{11} + 1{,}008 v_{21} - 14{,}688 v_{31} = 0 \quad / \cdot 23{,}8095$$

$$\{v_1\}^T [M] \{v_3\} = \{0\}$$

$$\{v_{11}\ v_{21}\ v_{31}\} \cdot \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \cdot \begin{Bmatrix} -0{,}710 \\ 1{,}0 \\ -0{,}507 \end{Bmatrix} = 0$$

$$-8{,}52 v_{11} + 24 v_{21} - 16{,}224 v_{31} = 0 \quad / \cdot 1{,}4084$$

$$v_{11} = 1{,}133\, v_{31}$$

$$v_{21} = 1{,}078\, v_{31}$$

Zapisano u matričnom obliku: $[S_2] = \begin{bmatrix} 0 & 0 & 1{,}133 \\ 0 & 0 & 1{,}078 \\ 0 & 0 & 1{,}0 \end{bmatrix}$

$$[\bar{D}_3] = [D] \cdot [S_2] = \begin{bmatrix} 0 & 0 & 142{,}56 \\ 0 & 0 & 136{,}72 \\ 0 & 0 & 125{,}25 \end{bmatrix}$$

Iteracija $[D_3]\cdot\{v_1\} = \lambda\{v_1\}$

			1	1,0	1,0
			1	0,959	0,959
			1	0,879	0,879
0	0	142,56	142,56	125,25	
0	0	136,72	136,72	120,119	-//-
0	0	125,25	125,25	110,042	
			142,56	125,25	125,25

Prva vlastita frekvencija $\lambda_3 = \omega_1^2 = 125,25 \rightarrow \omega_1 = 11,19$

Prvi vlastiti vektor $\{v_1\} = \begin{Bmatrix} 1,0 \\ 0,959 \\ 0,879 \end{Bmatrix}$

MATRIČNA ITERACIJA PREKO MATRICE FLEKSIBILNOSTI

$$[M]\{\ddot{x}\} + [k]\{x\} = \{0\}$$

$$\{x\} = \{v\} \cdot \sin \omega t$$

$$\{\ddot{x}\} = -\omega^2 \{v\} \sin \omega t$$

$$([k] - \omega^2 [M])\{v\} = \{0\} \mid \cdot [k]^{-1}$$

$$([I] - \omega^2 \underbrace{[k]^{-1} \cdot [M]})\{v\} = \{0\}$$

pomaci od in. sila

$$[F] \cdot [M] \cdot \{v\} = \frac{1}{\omega^2}\{v\}$$

$$[D] \cdot \{v\} = \lambda \cdot \{v\}$$

Prvi oblik

$$[D] = [F][M] = \begin{bmatrix} 1,9066 & 3,0417 & 3,5556 \\ 1,5208 & 3,0417 & 3,5556 \\ 1,3333 & 2,6667 & 3,5556 \end{bmatrix} \cdot 10^{-3}$$

	1	1	1	1
	1	0,955	0,952	0,951
	1	0,888	0,883	0,883
1,9066 3,0417 3,5556	8,50	7,97	7,94	7,94
1,5208 3,0417 3,5556	8,12	7,58	7,56	7,55
1,3333 2,6667 3,5556	7,56	7,04	7,01	7,01
	8,50	7,97	7,94	7,94

Prva vlastita frekvencija $\lambda_1 = \frac{1}{\omega_1^2} = 7,94 \cdot 10^{-3} \rightarrow \omega_1^2 = 125,94$

$\omega_1 = 11,22$ r/s

Prvi vlastiti oblik oscilacija $v_1 = \begin{Bmatrix} 1 \\ 0,951 \\ 0,883 \end{Bmatrix}$

Drugi vlastiti oblik

Uvjet ortogonalnosti 1. vektora $\{v_1\} \perp \{v_2\} = 0$

$$\{v_1\}^T [M] \{v_2\} = 0$$

$$\{1 \quad 0{,}951 \quad 0{,}883\} \cdot \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \cdot \begin{Bmatrix} v_{12} \\ v_{22} \\ v_{32} \end{Bmatrix} = 0$$

$$v_{12} = -1{,}902\, v_{22} - 2{,}355\, v_{32}$$

$$[S_1] = \begin{bmatrix} 0 & -1{,}902 & -2{,}355 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[D_2] = [D] \cdot [S_1] = \begin{bmatrix} 0 & -5{,}8476 & -9{,}3457 \\ 0 & 1{,}4904 & -0{,}2601 \\ 0 & 1{,}3067 & 4{,}1556 \end{bmatrix} \cdot 10^{-4}$$

	1	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0
	1	0,5	0,204	0,107	0,071	0,057	0,051	0,049	0,048
	-1	-0,814	-0,583	-0,507	-0,478	-0,467	-0,463	-0,461	-0,460
0 -5,8476 -9,3457	3,498	4,685	4,251	4,109	4,056	4,035	4,027	4,024	4,022
0 1,4904 -0,2601	1,751	0,958	0,456	0,292	0,230	0,206	0,197	0,193	0,191
0 1,3067 4,1556	-2,849	-2,730	-2,155	-1,966	-1,895	-1,868	-1,857	-1,852	-1,850
	3,498	4,685	4,251	4,109	4,056	4,035	4,027	4,024	4,022

Druga vlastita frekvencija $\lambda_2 = \frac{1}{\omega_2^2} = 4{,}022 \cdot 10^{-4} \rightarrow \omega_2 = 49{,}86$

Drugi vlastiti vektor $v_2 = \begin{Bmatrix} 1{,}0 \\ 0{,}048 \\ -0{,}460 \end{Bmatrix}$

Treći oblik

→ uvjet ortogonalnosti vlastitih vektora $\{v_1\} \perp \{v_3\}$ i $\{v_2\} \perp \{v_3\}$

$$\{v_1\}^T [M] \{v_3\} = 0$$

$$\{1 \quad 0{,}951 \quad 0{,}883\} \cdot \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \begin{Bmatrix} v_{13} \\ v_{23} \\ v_{33} \end{Bmatrix} = 0$$

$$12 v_{13} + 22{,}824 v_{23} + 28{,}256 v_{33} = 0$$

$$\{v_2\}^T [M] \{v_3\} = 0$$

$$\{1 \quad 0{,}048 \quad -0{,}460\} \cdot \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \begin{Bmatrix} v_{13} \\ v_{23} \\ v_{33} \end{Bmatrix} = 0$$

$$12 v_{13} + 1{,}152 v_{23} - 14{,}72 v_{33} = 0 \qquad / \cdot (-1) \;//\; (-19{,}8125)$$

$$v_{23} = -1{,}983 \, v_{33}$$

$$v_{13} = 1{,}417 \, v_{33}$$

$$[S_2] = \begin{bmatrix} 0 & 0 & 1{,}417 \\ 0 & 0 & -1{,}983 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[D_3] = [\Delta][S_1] = \begin{bmatrix} 0 & 0 & 2{,}256 \\ 0 & 0 & -3{,}210 \\ 0 & 0 & 1{,}569 \end{bmatrix} \cdot 10^{-4}$$

		1	−0,703	−0,703
		−1	1,0	1,0
		1	−0,489	−0,489
0 0 2,256	2,256		−1,10	−1,10
0 0 −3,210	3,210		1,57	1,57
0 0 1,569	1,569		−0,77	−0,77
	3,210		1,57	1,57

Treća frekvencija $\lambda_3 = \frac{1}{\omega_3^2} = 1{,}57 \cdot 10^{-4} \quad \omega_3 = 79{,}81 \text{ r/s}$

VLASTITI OBLICI OSCILACIJA

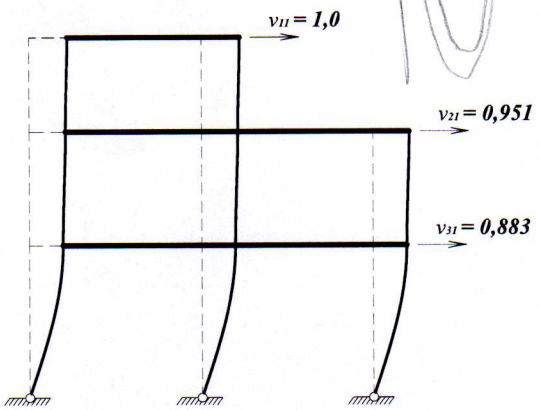
(Izračunati iteracijskim postupkom preko matrice fleksibilnosti)

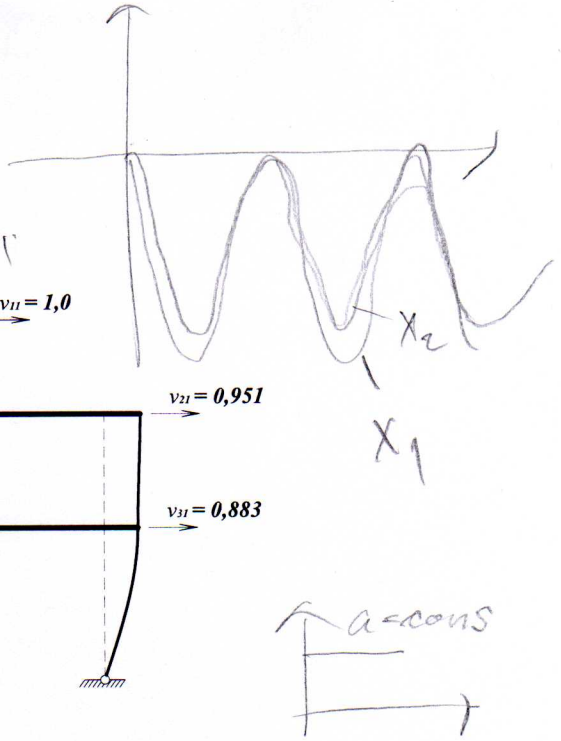
Prvi oblik:

$$\omega_1 = 11,22\ r/s$$

$$T_1 = \frac{2\pi}{\omega_1} = 0,56\ s$$

$$\{v_1\} = \begin{Bmatrix} 1,0 \\ 0,951 \\ 0,883 \end{Bmatrix}$$



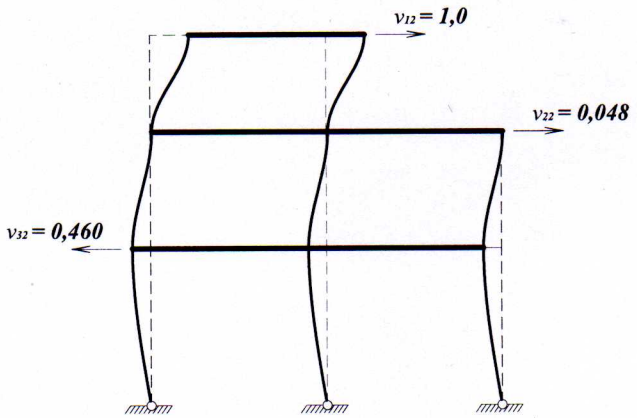


Drugi oblik:

$$\omega_2 = 49,86\ r/s$$

$$T_2 = \frac{2\pi}{\omega_2} = 0,126\ s$$

$$\{v_2\} = \begin{Bmatrix} 1,0 \\ 0,048 \\ -0,460 \end{Bmatrix}$$

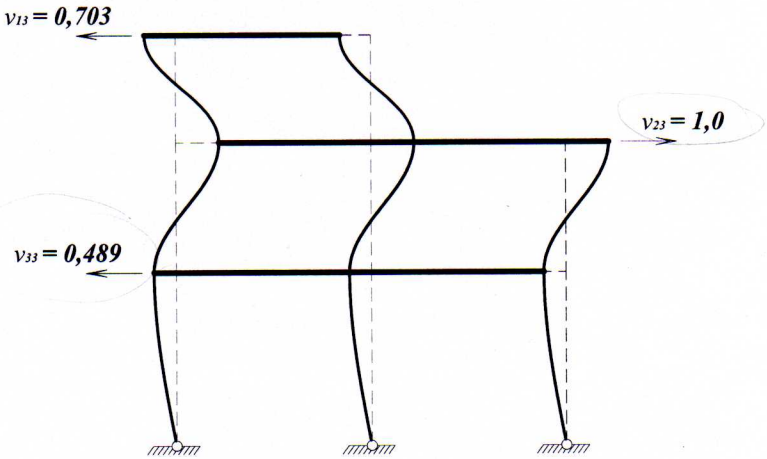


Treći oblik:

$$\omega_3 = 79,81\ r/s$$

$$T_3 = \frac{2\pi}{\omega_3} = 0,079\ s$$

$$\{v_3\} = \begin{Bmatrix} -0,703 \\ 1,0 \\ -0,489 \end{Bmatrix}$$



$$[M] = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 32 \end{bmatrix} \qquad [k] = \begin{bmatrix} 31104 & -31104 & 0 \\ -31104 & 95104 & -64000 \\ 0 & -64000 & 73000 \end{bmatrix}$$