Determinante

$$A = [a_{ij}]_{n\times n} \quad \Rightarrow |A| = |[a_{ij}]_{n\times n}| = \det A = D$$

Zadaci:

1. Izračunati:

a) $\begin{vmatrix} 8 & 3 \\ 7 & 5 \end{vmatrix}$; b) $\begin{vmatrix} 8 & 4 \\ 6 & 3 \end{vmatrix}$; c) $\begin{vmatrix} a-b & -2 \\ ab & a-b \end{vmatrix}$;

d) $\begin{vmatrix} 1 & \log_a b \\ \log_b a & 1 \end{vmatrix}$; e) $\begin{vmatrix} \operatorname{tg}\varphi & -1 \\ 1 & \operatorname{ctg}\varphi \end{vmatrix}$; f) $\begin{vmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{vmatrix}$;

g) $\begin{vmatrix} \cos\alpha + i\sin\alpha & 1 \\ 1 & \cos\alpha - i\sin\alpha \end{vmatrix}$; h) $\begin{vmatrix} 2i & -i \\ i & 3i \end{vmatrix}$.

2. Izračunati vrijednost determinanti:

a) $\begin{vmatrix} 3 & 5 & 1 \\ 1 & 4 & 2 \\ 2 & 0 & -1 \end{vmatrix}$; b) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & -6 \\ -1 & 2 & 6 \end{vmatrix}$; c) $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & -2 \\ -1 & 2 & 4 \end{vmatrix}$;

d) $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & a \end{vmatrix}$; e) $\begin{vmatrix} 1 & 0 & 1+i \\ 0 & 1 & i \\ 1-i & -1 & 1 \end{vmatrix}$.

3. Provjeriti

a) $\begin{vmatrix} b & a & a & a \\ a & b & a & a \\ a & a & b & a \\ a & a & a & b \end{vmatrix} = (b-a)(3a+b)$; b) $\begin{vmatrix} 4 & -1 & 1 & 5 \\ -3 & 5 & 1 & 2 \\ -2 & 2 & 1 & 1 \\ 2 & 5 & 1 & 1 \end{vmatrix} = -90$.

4. Riješiti jednadžbe:

a) $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$; b) $\begin{vmatrix} 4 & 8 & x+3 \\ x+2 & 6 & 3 \\ 1 & 2x & x \end{vmatrix} = 0$.

5. Riješiti jednadžbe:

a) $\begin{vmatrix} \sin\left(x+\frac{\pi}{4}\right) & \sin x & \cos x \\ \sin\left(x+\frac{\pi}{4}\right) & \cos x & \sin x \\ 1 & a & 1-a \end{vmatrix} = 0$; b) $\begin{vmatrix} 1 & 1 & x & 1 \\ 1 & -1 & -1 & -1 \\ x & 1 & 5 & 3 \\ 1 & 5 & 11 & 8 \end{vmatrix} = 0$.

6. Izračunati:

$$D = \begin{vmatrix} z & -z & 0 \\ 0 & z^2 & -1 \\ 1 & z & z+1 \end{vmatrix};$$ ako z zadovoljava jednadžbu $z^5 = 1$.

7. Riješiti sustave linearnih jednadžbi:

a) $\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y - 6z &= -2 \\ x - 2y - 6z &= 4 \end{aligned}$ b) $\begin{aligned} x + y + z &= 9 \\ x + 2y + 3z &= 16 \\ x + 3y + 4z &= 21 \end{aligned}$

c) $\begin{aligned} x + 2y - 14z &= -8 \\ 2x + 5y + 7z &= 9 \\ 4x - 2y - 3z &= 24 \end{aligned}$ d) $\begin{aligned} a + b + c &= -2 \\ 8a + 4b + 2c &= -4 \\ 27a + 9b + 3c &= 36 \end{aligned}$

8. Diskutirati sustav:

a) $\begin{aligned} 4x + 8y + (m+3)z &= -2 \\ (m+2)x + 6y + 3z &= 1 \\ x + 2my + mz &= -1 \end{aligned}$

b) $\begin{aligned} x + (m+2)y - z &= 0 \\ (m+2)x + y - z &= 1 \\ x + y - (m+2)z &= m+3 \end{aligned}$

9. Riješiti homogene sustave:

a) $\begin{aligned} 2x - y + 3z &= 0 \\ x + 2y - 5z &= 0 \\ 3x + y - 2z &= 0 \end{aligned}$ b) $\begin{aligned} 3x + 2y - z &= 0 \\ 2x - y + 3z &= 0 \\ x + 3y - 4z &= 0 \end{aligned}$

c) $\begin{aligned} 3x - y + 2z &= 0 \\ 2x + 3y - 5z &= 0 \\ x + y + z &= 0 \end{aligned}$

10. Odrediti parametar k tako da homogeni sustav ima i netrivijalna rješenja i naći ta rješenja.

a) $$\begin{array}{rcrcrcr} x & + & y & + & z & = & 0 \\ kx & + & 4y & + & z & = & 0 \\ 6x & + & (k+2)y & + & 2z & = & 0 \end{array}$$

b) $$\begin{array}{rcrcrcr} x & + & y & + & kz & = & 0 \\ x & - & y & - & z & = & 0 \\ kx & + & y & + & 5z & = & 0 \end{array}$$

11. Riješiti sustave jednadžbi:

a) $$\begin{array}{rcrcrcrcr} x & + & y & + & z & + & t & = & 5 \\ 2x & - & y & + & 4z & + & 5t & = & 8 \\ x & + & 2y & - & z & + & t & = & -2 \\ 3x & + & 2y & + & 2z & + & 2t & = & 8 \end{array}$$

b) $$\begin{array}{rcrcrcrcr} 2x & + & 3y & - & z & - & t & = & 0 \\ x & - & y & - & 2z & - & 4t & = & 0 \\ 3x & + & y & + & 3z & - & 2t & = & 0 \\ 6x & + & 3y & & & - & 7t & = & 0 \end{array}$$

12. Odrediti polinome $P_n(x)$ najnižeg stupnja koji ispunjavaju uvjete:
a) $P(2) = -3$, $P(-1) = 9$, $P(1) = -1$;
b) $P(1) = 4$, $P(2) = 3$, $P(-1) = 0$, $P(3) = 4$.

13. Graf funkcije $y = P(x)$, gdje je $P(x)$ polinom, prolazi kroz točke $M_1(1, -1)$, $M_2(0, 1)$, $M_3(2, 5)$ i $M_4(3, 37)$. Odrediti polinom $P(x)$.