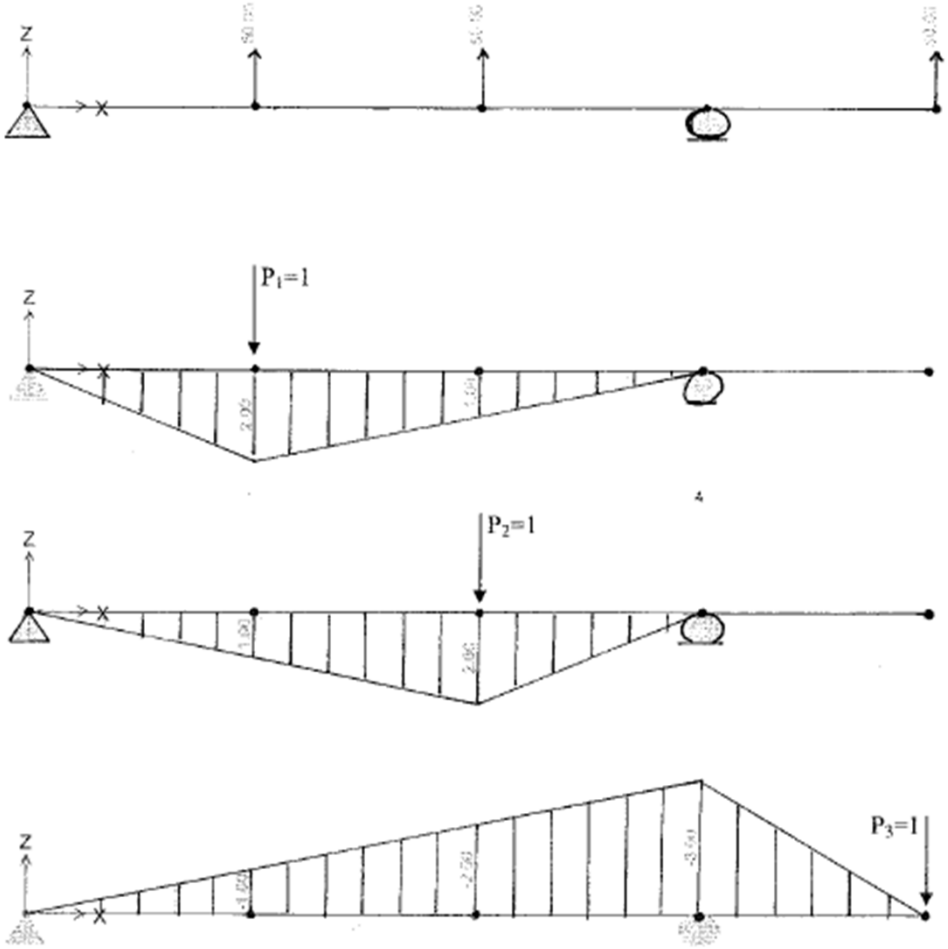
Dinamika konstrukcija i potresno inženjerstvo

Vježbe br.8 **21.05.12.**

Zadatak br.1

Za sustav prikazan na slici odrediti vlastite vrijednosti.

Poznato je: E=31,5GPa; b/d=0,3/0,9m; m=50t; L=4x3,0m.



$E=31{,}50\ \text{GPa} =$

$=31{,}5\cdot 10^{6}\ \text{kN/m}^2$

$I= 0{,}3 \cdot 0{,}9^3/12 =$

$= 1{,}8225\cdot 10^{-3}\ \text{m}^4$

Matrica fleksibilnosti:

$$[D] = \frac{1}{EI}\begin{bmatrix} 12{,}0 & 10{,}5 & -12{,}5 \\ 10{,}5 & 12{,}0 & -15{,}0 \\ -12{,}0 & -15{,}0 & 36{,}0 \end{bmatrix}$$

Matrica masa:

$$[M] = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Umnožak matrica [D] i [M] = **dinamička matrica** $[D_M]$:

$$[D_M] = [D][M] = \frac{m}{EI}\begin{bmatrix} 12{,}0 & 10{,}5 & -12{,}5 \\ 10{,}5 & 12{,}0 & -15{,}0 \\ -12{,}0 & -15{,}0 & 36{,}0 \end{bmatrix}$$

<u>Vektorska iteracija</u>

U vektorskoj iteraciji se koristi postupak:

$$[D_M]\{\emptyset\} = \frac{1}{\omega^2}\{\emptyset\}$$

Prvi (osnovni) oblik (ton, mod)

Pretpostavlja se oblik osciliranja prvog tona.

$$\emptyset_1^* = \begin{bmatrix} 1{,}0 \\ 1{,}2 \\ -2{,}0 \end{bmatrix};\ p^* = \frac{m}{EI}$$

U narednim koracima i tablici prikazan je iterativni postupak gdje je amplituda Φ_1 uvijek reducirana na jedinicu.

1.Korak:
$$D_M \begin{bmatrix} 1{,}0 \\ 1{,}2 \\ -2{,}0 \end{bmatrix} = p^* \begin{bmatrix} 48{,}6 \\ 54{,}9 \\ -102{,}0 \end{bmatrix} = 48{,}6p^* \begin{bmatrix} 1{,}0 \\ 1{,}1296 \\ -2{,}0988 \end{bmatrix}$$

2.Korak:
$$D_M \begin{bmatrix} 1{,}0 \\ 1{,}1296 \\ -2{,}0988 \end{bmatrix} = p^* \begin{bmatrix} 49{,}0464 \\ 55{,}5372 \\ -104{,}5008 \end{bmatrix} = 49{,}0464p^* \begin{bmatrix} 1{,}0 \\ 1{,}1323 \\ -2{,}1307 \end{bmatrix}$$

3.Korak:
$$D_M \begin{bmatrix} 1{,}0 \\ 1{,}1323 \\ -2{,}1307 \end{bmatrix} = p^* \begin{bmatrix} 49{,}4575 \\ 56{,}0481 \\ -105{,}6897 \end{bmatrix} = 49{,}4575p^* \begin{bmatrix} 1{,}0 \\ 1{,}1333 \\ -2{,}1370 \end{bmatrix}$$

4.Korak:

$$D_M \begin{bmatrix} 1{,}0 \\ 1{,}1333 \\ -2{,}1370 \end{bmatrix} = p^* \begin{bmatrix} 49{,}5437 \\ 56{,}1546 \\ -105{,}9315 \end{bmatrix} = 49{,}5437p^* \begin{bmatrix} 1{,}0 \\ 1{,}1334 \\ -2{,}1381 \end{bmatrix}$$

5.Korak:

$$D_M \begin{bmatrix} 1{,}0 \\ 1{,}1334 \\ -2{,}1381 \end{bmatrix} = p^* \begin{bmatrix} 49{,}5579 \\ 56{,}1723 \\ -105{,}9726 \end{bmatrix} = 49{,}5579p^* \begin{bmatrix} 1{,}0 \\ 1{,}1335 \\ -2{,}1384 \end{bmatrix}$$

6.Korak:

$$D_M \begin{bmatrix} 1{,}0 \\ 1{,}1335 \\ -2{,}1384 \end{bmatrix} = p^* \begin{bmatrix} 49{,}5626 \\ 56{,}1780 \\ -105{,}9849 \end{bmatrix} = 49{,}5626p^* \begin{bmatrix} 1{,}0 \\ 1{,}1335 \\ -2{,}1384 \end{bmatrix}$$

Rezultati iteracija dani su u tablici:

ITERACIJA	$\lambda_{(1)}$	Φ_{11}	Φ_{21}	Φ_{31}
1	48,6p*	1,0	1,1296	-2,0988
2	49,0464p*	1,0	1,1323	-2,1307
3	49,4575p+	1,0	1,1333	-2,1370
4	49,5437p*	1,0	1,1334	-2,1381
5	49,5579p*	1,0	1,1335	-2,1384
6	49,5626p*	1,0	1,1335	-2,1384

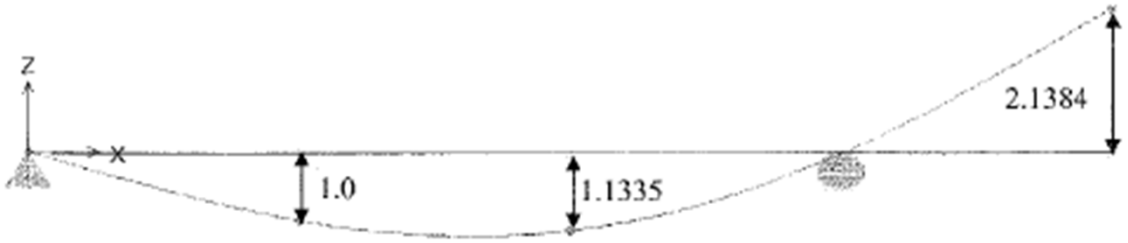
Rješenje je stabilizirano, pa je prema tome:

$$\lambda_1 = 49{,}5626\frac{m}{EI} = \frac{1}{\omega_1^2} \rightarrow \omega_1 = \sqrt{\frac{EI}{49{,}5626m}} = 15{,}2204 rad/s$$

$$T_1 = 0{,}4128 s$$

Prvi ton je (normaliziran):

$$\{\Phi_1\} = \begin{Bmatrix} 1{,}0 \\ 1{,}1335 \\ -2{,}1384 \end{Bmatrix}$$



Drugi vlastiti oblik

Pretpostaviti oblik osciliranja u drugom tonu: $\emptyset_2^* = \begin{bmatrix} 1{,}0 \\ 0{,}8 \\ 1{,}5 \end{bmatrix}$

Na probni stupac $\{\Phi\}$ primjenjujemo sljedeći uvjet ortogonalnosti:

$$\{\Phi_1\}^T[M]\{\Phi_2\} = \{\Phi_{1,1} \quad \Phi_{2,1} \quad \Phi_{3,1}\}\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \emptyset_{1,2} \\ \emptyset_{2,2} \\ \emptyset_3, 2 \end{Bmatrix} = 0$$

Ili u razvijenom obliku (vežemo probnu vrijednost Φ_1 , dok su Φ_2 i Φ_3 proizvoljni):

$$\Phi_1^T M\Phi_2 = \Phi_{1,1}m_1\Phi_{1,2} + \Phi_{2,1}m_2\Phi_{2,2} + \Phi_{3,1}m_3\Phi_{3,2} = 0$$

$$1.0m\Phi_{1,2} + 1.1335m\Phi_{2,2} - 2.1384m\Phi_{3,2} = 0$$

$$\Phi_{1,2} = 0.0\Phi_{1,2} - 1.1335\Phi_{2,2} + 2.1384\Phi_{3,2}$$

$$\Phi_{2,2} = 0.0\Phi_{1,2} + 1.0\Phi_{2,2} + 0.0\Phi_{3,2}$$

$$\Phi_{3,2} = 0.0\Phi_{1,2} + 0.0\Phi_{2,2} + 1.0\Phi_{3,2}$$

U matričnom obliku:

$$S_1 = \begin{bmatrix} 0 & -1{,}1335 & 2{,}1384 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Prethodna kvadratna matrica se zove matrica čišćenja ili eliminacije [S], koja nakon množenja s dinamičkom matricom $[D_M]$ daje matricu $[D]_1$, koja se zatim koristi u iterativnom postupku za drugi vlastiti oblik.

$$D_{M1} = D_M S_1 = \frac{m}{EI}\begin{bmatrix} 0 & -1{,}1335 & 2{,}1384 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Grubo procijeniti oblik drugog tona.

Iterativni postupak je sada isti kao i kod prvog tona, s tim što sada koristimo vezanu dinamičku matricu:

$$[D]_{M1} = [D_M][S_1]$$

1.Korak iterativni

$$D_{M1}\begin{bmatrix} 1.0 \\ 0.8 \\ 1.5 \end{bmatrix} = p^*\begin{bmatrix} 18.0096 \\ 11.2584 \\ 14.3904 \end{bmatrix} = 18.0096\,p^*\begin{bmatrix} 1.0 \\ 0.6251 \\ 0.7990 \end{bmatrix}$$

2.Korak

$$D_{M1}\begin{bmatrix} 1.0 \\ 0.6251 \\ 0.7990 \end{bmatrix} = p^*\begin{bmatrix} 8.9759 \\ 6.0165 \\ 7.3871 \end{bmatrix} = 8.9759\,p^*\begin{bmatrix} 1.0 \\ 0.6703 \\ 0.8230 \end{bmatrix}$$

3,4,5,i 6.ti korak iteracije

$$D_{M1}\begin{bmatrix}1.0\\0.6703\\0.8230\end{bmatrix}=p^{*}\begin{bmatrix}9.1636\\6.1998\\7.5721\end{bmatrix}=9.1636p^{*}\begin{bmatrix}1.0\\0.6766\\0.8263\end{bmatrix}$$

$$D_{M1}\begin{bmatrix}1.0\\0.6766\\0.8263\end{bmatrix}=p^{*}\begin{bmatrix}9.1891\\6.2251\\7.5974\end{bmatrix}=9.1891p^{*}\begin{bmatrix}1.0\\0.6774\\0.8268\end{bmatrix}$$

$$D_{M1}\begin{bmatrix}1.0\\0.6774\\0.8268\end{bmatrix}=p^{*}\begin{bmatrix}9.1935\\6.2289\\7.6014\end{bmatrix}=9.1935p^{*}\begin{bmatrix}1.0\\0.6775\\0.8268\end{bmatrix}$$

$$D_{M1}\begin{bmatrix}1.0\\0.6775\\0.8268\end{bmatrix}=p^{*}\begin{bmatrix}9.1931\\6.2289\\7.6013\end{bmatrix}=9.1931p^{*}\begin{bmatrix}1.0\\0.6776\\0.8268\end{bmatrix}$$

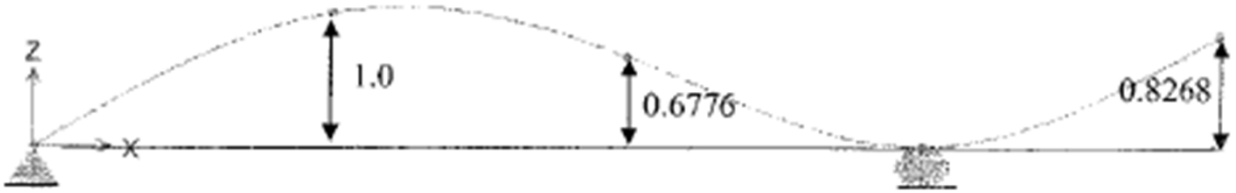
7. i 8. korak iteracije

$$D_{M1}\begin{bmatrix}1.0\\0.6776\\0.8268\end{bmatrix}=p^{*}\begin{bmatrix}9.1928\\6.2289\\7.6012\end{bmatrix}=9.1928p^{*}\begin{bmatrix}1.0\\0.6776\\0.8269\end{bmatrix}$$

$$D_{M1}\begin{bmatrix}1.0\\0.6776\\0.8269\end{bmatrix}=p^{*}\begin{bmatrix}9.1942\\6.2296\\7.6022\end{bmatrix}=9.1942p^{*}\begin{bmatrix}1.0\\0.6776\\0.8268\end{bmatrix}$$

$$9.1942\frac{m}{EI}=\frac{1}{\omega_2^2}\Rightarrow\omega_2=\sqrt{\frac{EI}{9.1942m}}=35.3411rad/s$$

$$T_1=0.1778s\qquad\Phi_1=\begin{bmatrix}1.0\\0.6776\\0.8268\end{bmatrix}$$



Tablični prikaz iteracija:

ITERACIJA	$\lambda_{(2)}$	Φ_{12}	Φ_{22}	Φ_{32}
1	18,0096p*	1,0	0,6251	0,7990
2	8,9759p*	1,0	0,6703	0,8230
3	9,1636p*	1,0	0,6766	0,8263
4	9,1891p*	1,0	0,6774	0,8268
5	9,1935p*	1,0	0,6775	0,8268
6	9,1931p*	1,0	0,6776	0,8268
7	9,1928p*	1,0	0,6776	0,8269
8	9,1942p*	1,0	0,6776	0,8268

Treći vlastiti oblik (ton)

Za određivanje trećeg tona koriste se 2 uvjeta ortogonalnosti:

$$\Phi_1^T M\Phi_3 = 0 \Rightarrow [\Phi_{1,1}\ \Phi_{2,1}\ \Phi_{3,1}]\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \Phi_{1,3} \\ \Phi_{2,3} \\ \Phi_{3,3} \end{bmatrix} = 0$$

$$\Phi_2^T M\Phi_3 = 0 \Rightarrow [\Phi_{1,2}\ \Phi_{2,2}\ \Phi_{3,2}]\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \Phi_{1,3} \\ \Phi_{2,3} \\ \Phi_{3,3} \end{bmatrix} = 0$$

$$1.0m\Phi_{1,3} + 1.1335m\Phi_{2,3} - 2.1384m\Phi_{3,3} = 0 \Rightarrow \Phi_{1,3} = -1.1335\Phi_{2,3} + 2.1384\Phi_{3,3}$$

$$1.0m\Phi_{1,3} + 0.6776m\Phi_{2,3} + 0.8268m\Phi_{3,3} = 0 \Rightarrow$$

$$\Rightarrow -1.1335\Phi_{2,3} + 2.1384\Phi_{3,3} + 0.6776\Phi_{2,3} + 0.8268\Phi_{3,3} = 0 \Rightarrow$$

$$\Rightarrow -0.4559\Phi_{2,3} + 2.9652\Phi_{3,3} = 0 \Rightarrow \quad \Phi_{2,3} = 0.0\Phi_{1,3} + 0.0\Phi_{2,3} + 6.5041\Phi_{3,3}$$

$$\Phi_{1,3} = 0.0\Phi_{1,3} + 0.0\Phi_{2,3} - 5.2340\Phi_{3,3}$$

$$\Phi_{3,3} = 0.0\Phi_{1,3} + 0.0\Phi_{2,3} + \Phi_{3,3}$$

Matrica čišćenja ili eliminacije je:

$$[S]_2 = \begin{bmatrix} 0 & 0 & -5{,}2340 \\ 0 & 0 & 6{,}5041 \\ 0 & 0 & 1{,}00 \end{bmatrix}$$

Modificirana dinamička matrica za dalje iteracije je :

$$[D]_{M2} = [D_M][S]_2 = \frac{m}{EI}\begin{bmatrix} 0 & 0 & -6{,}5149 \\ 0 & 0 & 8{,}0922 \\ 0 & 0 & 1{,}2464 \end{bmatrix}$$

Iteracija:

1.Korak

$$D_{M2}\begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = p^*\begin{bmatrix} -6.5149 \\ 8.0922 \\ 1.2464 \end{bmatrix} = 6.5149\,p^*\begin{bmatrix} -1.0 \\ 1.2421 \\ 0.1913 \end{bmatrix}$$

2.korak

$$D_{M2}\begin{bmatrix} -1.0 \\ 1.2421 \\ 0.1913 \end{bmatrix} = p^*\begin{bmatrix} -1.2463 \\ 1.5480 \\ 0.2385 \end{bmatrix} = 1.2463\,p^*\begin{bmatrix} -1.0 \\ 1.2421 \\ 0.1913 \end{bmatrix}$$

3.korak

$$D_{M2}\begin{bmatrix}-1.0\\1.2421\\0.1913\end{bmatrix}=p^{*}\begin{bmatrix}-1.2463\\1.5480\\0.2385\end{bmatrix}=1.2463p^{*}\begin{bmatrix}-1.0\\1.2421\\0.1913\end{bmatrix}$$

$$1.2463\frac{m}{EI}=\frac{1}{\omega_2^2}\Rightarrow\omega_2=\sqrt{\frac{EI}{1.2463m}}=95.9827rad/s$$

$$T_1=0.0655s\qquad\Phi_3=\begin{bmatrix}1.0\\-1.2421\\-0.1913\end{bmatrix}$$