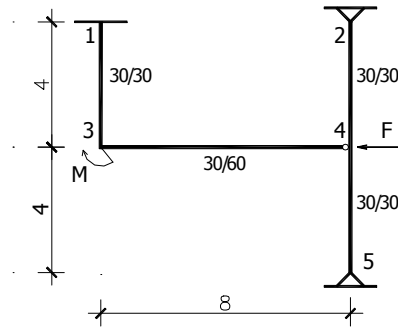
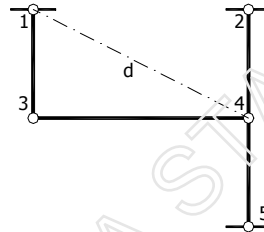


2 Tehničkom metodom pomaka riješiti dani sustav, te nacrtati M dijagram.

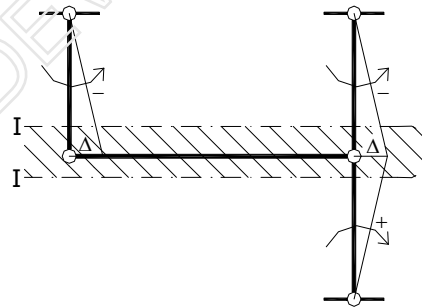


Opterećenje koje djeluje na konstrukciju je $F=20\text{kN}$ i $M=20\text{kNm}$. Momenti tromosti greda i stubova zadani su na crtežu preko odnosa visine i širine poprečnog presjeka b/h , dok je Youngov modul elastičnosti $E=\text{const}$.

Zglobna shema ovog sustava je:



a shema pomjeranja izgleda:



tako da su nepoznate:

-kutovi zaokreta φ_3, φ_4 i pomjeranje Δ .

Odredimo sada krutosti pojedinih članova:

$$I_G = \frac{0,3 \cdot 0,6^3}{12}, I_S = \frac{0,3 \cdot 0,3^3}{12} \Rightarrow \frac{I_G}{I_S} = 8 \Rightarrow I_G = 8 \cdot I_S$$

$$K_{\text{greda}} = \frac{2EI}{l} = \frac{2EI_G}{8} = \frac{2E \cdot 8I_S}{8} \cdot \frac{4}{4} = 4K$$

$$K_{\text{stupa}} = \frac{2EI}{l} = \frac{2EI_S}{4} = K$$

Postavimo jednadžbe iz kojih odredimo nepoznate kutove zaokreta φ_3, φ_4 i pomjeranje Δ .

$$\varphi_3 \longrightarrow \sum M_3 = 20$$

$$\varphi_4 \longrightarrow \sum M_4 = 0$$

$$\Delta \longrightarrow \sum X^{I-1} = 0$$

$$\sum M_3 = 20$$

$$M_{31} + M_{34} = 20$$

$$M_{31} = ?$$



$$\begin{aligned} M_{31} &= K_{31}(2\varphi_3 + \varphi_1 - 3\Psi_{31}) + \bar{M}_{31}^0 = \\ &= K(2\varphi_3 - 3\frac{(-\Delta)}{4}) = \\ &= 2K\varphi_3 + 0,75K\Delta \end{aligned}$$

$$M_{34} = ?$$



$$\begin{aligned} M_{34} &= K_{34}(1,5\varphi_3 - 1,5\Psi_{34}) + \bar{M}_{34}^0 = \\ &= 4K(1,5\varphi_3) = 6K\varphi_3 \end{aligned}$$

Nakon uvrštavanja, dobit ćemo prvu jednadžbu:

$$M_{31} + M_{34} = 20$$

$$2K\varphi_3 + 0,75K\Delta + 6K\varphi_3 = 20$$

$$8K\varphi_3 + 0,75K\Delta = 20$$

Drugi uvjet je:

$$\sum M_4 = 0$$

$$M_{42} + M_{45} = 0$$

$$M_{42} = ?$$



$$\begin{aligned} M_{42} &= K_{42}(1,5\varphi_4 - 1,5\Psi_{42}) + \bar{M}_{42}^0 = \\ &= K(1,5\varphi_4 - 1,5\frac{(-\Delta)}{4}) = \\ &= 1,5K\varphi_4 + 0,375K\Delta \end{aligned}$$

$$M_{45} = ?$$



$$\begin{aligned} M_{45} &= K_{45}(1,5\varphi_4 - 1,5\Psi_{45}) + \bar{M}_{45}^0 = \\ &= K(1,5\varphi_4 - 1,5\frac{\Delta}{4}) = \\ &= 1,5K\varphi_4 - 0,375K\Delta \end{aligned}$$

Druga jednadžba je:

$$M_{42} + M_{45} = 0$$

$$1,5K\varphi_4 + 0,375K\Delta + 1,5K\varphi_4 - 0,375K\Delta = 0$$

$$3K\varphi_4 = 0 \Rightarrow \varphi_4 = 0$$



$$\sum X^{I-1} = 0$$

$$\sum R_0 - \sum \frac{M_i + M_k}{l_{ik}} = T$$

$$R_0 = 0 \quad T = -20$$

$$\sum \frac{M_i + M_k}{l_{ik}} = -\frac{M_{31} + M_{13}}{4} - \frac{M_{42} + M_{24}}{4} + \frac{M_{45} + M_{54}}{4}$$

$$\begin{aligned} M_{13} &= K_{13}(2\varphi_1 + \varphi_3 - 3\Psi_{13}) + \bar{M}_{13}^0 = \\ &= K(\varphi_3 - 3\frac{(-\Delta)}{4}) = K\varphi_3 + 0,75K\Delta \end{aligned}$$

$$\sum \frac{M_i + M_k}{l_{ik}} = \dots = -0,75K\varphi_3 + 0,5625K\Delta$$

Nakon uvrštavanja dobivamo tri jednadžbu:

$$0,75K\varphi_3 + 0,5625K\Delta = -20$$

Budući smo iz druge jednadžbe dobili da je $\varphi_4=0$, sada dobijamo sustav dvije jednadžbe s dvije nepoznate:

$$\begin{aligned} 8K\varphi_3 + 0,75K\Delta &= 20 \\ 0,75K\varphi_3 + 0,5625K\Delta &= -20 \end{aligned}$$

Rješenje sustava: $\Delta = -\frac{44,44}{K} \quad \varphi_3 = \frac{6,67}{K}$

Vrijednosti momenata su:

$$M_{13} = \dots = -26,667 \text{ kNm}$$

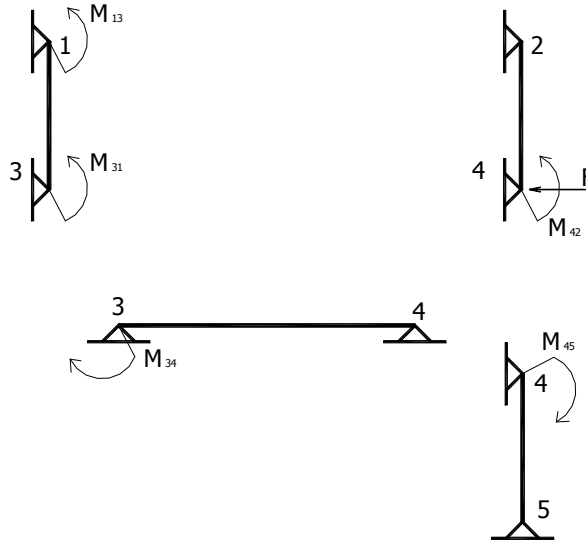
$$M_{31} = \dots = -20 \text{ kNm}$$

$$M_{34} = \dots = 40 \text{ kNm}$$

$$M_{42} = \dots = -16,665 \text{ kNm}$$

$$M_{45} = \dots = 16,665 \text{ kNm}$$

Dijagram M crtamo na osnovu presječnih veličina dobivenih rješavanjem sustava prostih greda opterećenih momentima (nanosimo ih na grede u skladu sa predznacima usvojenim za ovu metodu) i vanjskim opterećenjem, a kao na donjem crtežu:



Dijagrami izgledaju:

